SOLUTION OF A NONSTEADY-STATE POROUS COOLING PROBLEM BY AN APPROXIMATE METHOD

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The solution of porous cooling problems by an approximate method using electronic computers is considered.

Porous materials are used most often in such constructions as heat exchangers, turbine buckets, etc. The extensive usage of these materials is associated with the possibility of effective cooling of the apparatus walls because of filtration of the fluid or gas through the porous body.

Questions of the analytical computation of the temperature fields during porous cooling and evaporative cooling through the pores have been examined in [1-5]. A solution of the steady-state problem of heat propagation in a plate during porous cooling is given in [1], where it is shown that the temperatures of the solid skeleton of material and the fluid being filtered are hardly different at any point of the porous structure. This result is quite essential since the analysis of the heat-transfer process under consideration can be simplified greatly. Results of an analytical solution of the nonsteady-state problem of computing the temperature fields of a porous plate are presented in [3].

It must be noted that existing analytical solutions of individual problems are not often successfully represented in a form convenient for practical utilization. Hence, we have solved the nonsteady-state porous cooling problem by an approximate method. The basic aim of such computations is to compare the results of the analytical solution of the nonsteady-state problem [3] with the results of the approximate method.

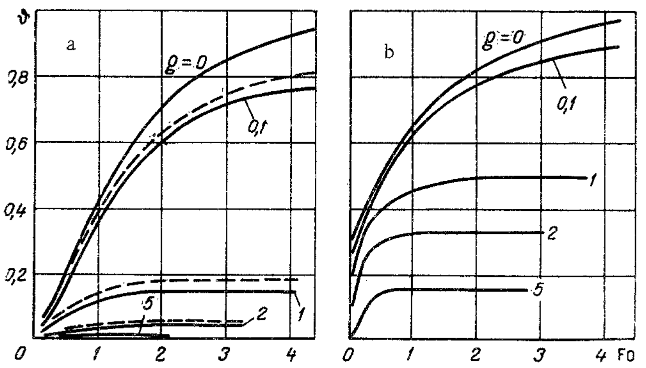

Fig. 1. Change in the temperature of a porous plate surface for Bi = 1: a) from the entrance of the coolant: solid lines) approximate method of computation; dashes) analytical computation; b) from the hot stream (the results of the approximate and analytic methods coincide).

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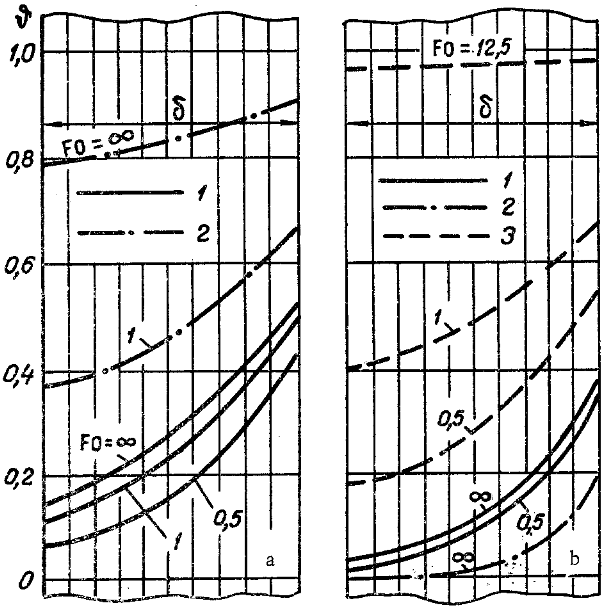

Fig. 2. Temperature profiles for Bi = 1. a: 1) g = 1; 2) 0.1; b: 1) g = 2; 2) 5; 3) 0.01.

The problem is formulated thus: initially, an infinite plate of finite thickness δ has the identical temperature T_i everywhere. A coolant (liquid or gas) in a reservoir at a temperature $T_l = T_i$ proceeds from the domain $x \ll 0$ into the plate at $x = 0$ and flows through the porous material at the constant mass flow rate G_l. Prior to the beginning of the porous cooling process the coolant in the reservoir and the porous wall are in thermal equilibrium at the temperature T_i. Then at the time $\tau = 0$ the plate is suddenly subjected to the effect of the surrounding gas medium with the constant temperature $T_m > T_i$ at $x > \delta$. The coefficient α of convective heat exchange from the hot gas to the plate surface is assumed constant and homogeneous over the surface for $x = \delta$.

The differential equation describing the nonsteady-state process of the change in temperature of the porous plate during cooling is for this problem

$$\lambda_s \frac{\partial^2 T}{\partial x^2} - c_l \frac{G_l}{F} \frac{\partial T}{\partial x} = c_s \rho_s \frac{\partial T}{\partial \tau}. \tag{1}$$

Here $T(x, \tau)$ should satisfy the following boundary and initial conditions:

$$\lambda_s \frac{\partial}{\partial x} T(0, \tau) = c_l G_l \left[T(0, \tau) - T_l\right] \frac{1}{F};$$

$$\lambda_s \frac{\partial}{\partial x} T(\delta, \tau) = \alpha \left[T_m - T(\delta, \tau)\right]; \tag{2}$$

$$T(x, 0) = T_i.$$

A general solution of (1), represented in dimensionless form in conformity with [3] will be

$$\vartheta = \left(\frac{\mathrm{Bi}}{\mathrm{Bi}+g}\right) \exp\left[-g(1-\xi)\right]$$

$$-4\left(\frac{\mathrm{Bi}}{\mathrm{Bi}+g}\right) \sum_{n=1}^{\infty} \frac{M_n^2 \sin M_n \exp\left[-\left(\frac{g^2}{4} + M_n^2\right)\mathrm{Fo} - \left(\frac{g}{2}\right)(1-\xi)\right]}{\left(\frac{g^2}{4} + M_n^2\right)\left[2M_n - \frac{(R+M_n^2)}{(R-M_n^2)} \sin 2M_n\right]}$$

$$\times \left(\frac{\frac{g}{2}}{M_n} \sin M_n \xi + \cos M_n \xi\right).$$

Here

$$R = \left(\frac{g}{2}\right)\left(\mathrm{Bi} + \frac{g}{2}\right),$$

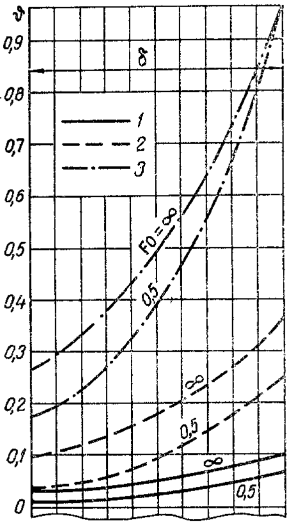

Fig. 3

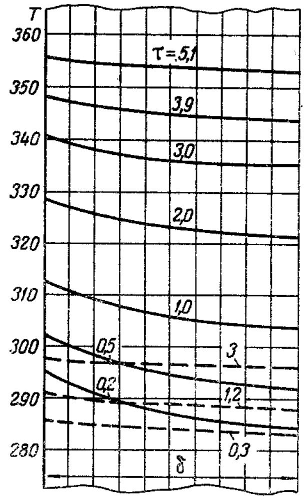

Fig. 4

Fig. 3. Temperature profiles for g = 1: 1) Bi = 0.1; 2) 0.5; 3) 10.

Fig. 4. Temperature profiles for gas filtration: solid line) $G_l = 5 \cdot 10^{-3}$ kg/sec; dashes) $G_l = 1 \cdot 10^{-3}$ kg/sec; $T_{gas} = 373$°K; $T_{ini} = 283$°K; $\delta = 1 \cdot 10^{-3}$ m; $F = 0.58 \cdot 10^{-2}$ m^2; $\lambda_s = 2.8$ W/m · deg; $\varepsilon = 0.55$; $c_s = 0.531 \cdot 10^3$ J/kg · deg.

and the quantity M_n is determined from the equation

$$\frac{\left(M_n^2 - \frac{g^2}{4}\right)\operatorname{tg} M_n - gM_n}{\left(\frac{g}{2}\right)\operatorname{tg} M_n + M_n} = \mathrm{Bi}.$$

The method of elementary thermal balances is used to solve this problem. The process of temperature variation in the porous plate, as described by (1) with the boundary and initial conditions (2), can then be expressed by the following system of differential equations

$$(c_s G_s)\frac{dT_1}{d\tau} = \frac{\lambda_s(1-\Pi)F}{\Delta x}(T_2 - T_1) - c_l G_l (T_1 - T_l);$$

$$(c_s G_s)\frac{dT_2}{d\tau} = -\frac{\lambda_s(1-\Pi)F}{\Delta x}(T_2 - T_1) + \frac{\lambda_s(1-\Pi)F}{\Delta x}(T_3 - T_2) - c_l G_l (T_2 - T_1);$$

$$(c_s G_s)\frac{dT_n}{d\tau} = -\frac{\lambda_s(1-\Pi)F}{\Delta x}(T_n - T_{n-1}) + \frac{\lambda_s(1-\Pi)F}{\Delta x}(T_{n+1} - T_n) - c_l G_l (T_n - T_{n-1}); \qquad (3)$$

$$(c_s G_s)\frac{dT_{n+1}}{d\tau} = -\frac{\lambda_s(1-\Pi)F}{\Delta x}(T_{n+1} - T_n) + \alpha F(T_m - T_{n+1}) - c_l G_l (T_{n+1} - T_n).$$

Numerical integration of the system of differential equations obtained was carried out by the Euler method on a Minsk-22 electronic computer. The results of computations, expressed in dimensionless form, are represented in Figs. 1-3. The computed quantities obtained for different porous plate thicknesses and represented in dimensionless form are superposed exactly and agree well with the results of the analytic solution. Such correspondence is achieved because of the sufficiently high value of the ratio $\delta/\Delta x$ and the small value of the integration spacing $\Delta\tau$.

We also computed the filtration of hot dry air through a porous body into a vacuum. The temperature profiles obtained by the numerical solution of (1) under the following boundary and initial conditions

$$\lambda_s \frac{\partial}{\partial x} T(0, \tau) = c_l \frac{G_l}{F} [T(0, \tau) - T_l];$$

$$\lambda_s \frac{\partial}{\partial x} T(\delta, \tau) = \varepsilon C_0 \left(\frac{T(\delta, \tau)}{100} \right)^4; \qquad (4)$$

$$T(x, 0) = T_l.$$

are shown in Fig. 4. It must be noted that the solution of porous cooling problems by an approximate method by using an electronic computer affords the possibility of performing computations with a broad variation in the initial parameters.

NOTATION

T	is the temperature, °K;
c_l	is the specific heat of the fluid, J/kg · deg;
c_s	is the specific heat of the porous material, J/kg · deg;
λ_s	is the coefficient of heat conduction of the porous material, W/m · deg;
τ	is the time, sec;
δ	is the porous plate thickness, m;
α	is the coefficient of convective heat exchange, $W/m^2 \cdot deg$;
G_s	is the weight of the i-th layer of a porous element, kg;
F	is the area of the plate, m^2;
G_l	is the mass flow rate, kg/sec;
Π	is the porosity;
$\Delta x = \delta/n$	is the thickness of the i-th porous element, m;
ρ_s	is the density of the porous material, kg/m^3;
ε	is the emissivity of the porous material;
C_0	is the radiation coefficient of an absolutely black body, $W/m^2 \cdot deg^4$;
$\vartheta = (T - T_l)/(T_m - T_l)$	is the dimensionless temperature;
$\xi = x/\delta$	is the dimensionless running coordinate;
$a = \lambda_s/\rho_s c_s$	is the temperature conduction of the porous material;
$Fo = a\tau/\delta^2$	is the Fourier number;
$Bi = \alpha\delta/\lambda_s$	is the Biot number;
$g = c_l G_l \delta/F\lambda_l$	is the dimensionless fluid flow rate.

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